Western Cape Government
Education

Directorate: Curriculum FET

TECHNICAL SCIENCES

REVISION BOOKLET
2021 TERM 1

Grade 11

This revision program is designed to assist you in revising the critical content and skills covered during the 1st term. The purpose is to prepare you to understand the key concepts and to provide you with an opportunity to establish the required standard and the application of the knowledge necessary to succeed in the NCS examination.

The revision program for covers MECHANICS

CONTENT PAGE

CONTROL TESTS & EXAMINATIONS ***1***

READING TIME

In every examination, you will be afforded ten minutes reading time before you can start writing your answers. This time is crucial. Do not waste it by looking around, looking bored or even putting your head on your desk. The reading time allow you to see where you might have any difficulty and you can then decide which questions will be easier to answer and you might prefer to start with those particular questions, The reading time also allows your memory to “wake up” and that feeling of knowing nothing before an examination usually fades during this reading session and you feel much more confident

TOPIC 1: INTRODUCTION TO MECHANICS

The content below indicates the areas that you should master in order to be successful in control tests and examinations.

Sign conventions

- Use the Cartesian coordinate system to indicate directions (+x and +y as positive). The indication of direction by using one point relative to another in the Cartesian plane will not be addressed.
- Use compass directions to indicate direction. Recall that vectors have magnitude an direction. Express direction using bearing by measuring from the north line in the clockwise direction to the vector.
- Use the above methods to determine the directions of vectors.

Graphs

- Demonstrate direct proportion graphs in the context of technology. Recall that straight line graphs are represented by $y = mx + c$.
- Demonstrate inverse proportion graphs in the context of technology. Recall that hyperbolic graphs are represented by $xy = k$.

Theorem of Pythagoras

- Determine the resultant of two vectors acting perpendicular to each other using the theorem of Pythagoras. Use the theorem of Pythagoras to calculate the resultant of forces in the context of technology.

<u>Prior Knowledge: Scalars and vectors:</u>

<u>Scalar</u>	<u>Vector</u>
A scalar is a physical quantity which has magnitude (size) only.	A vector is defined as a physical quantity which has both magnitude and direction.  **F**

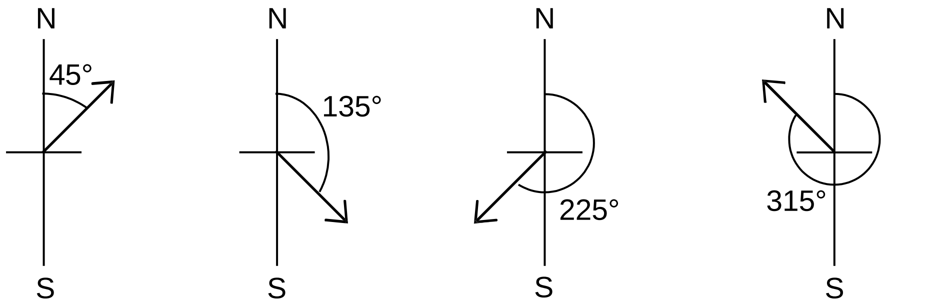
Important terms/definitions 2

Cartesian plane	A plane surface with the x-axis and y-axis intersecting at right angles.
Gradient	The ratio of change in y- coordinates to x-coordinates.
y-intercept	The point where a graph intersects the y-axis.

A Sign conventions

1 Quick facts: Bearing as a way to describe the direction of a vector

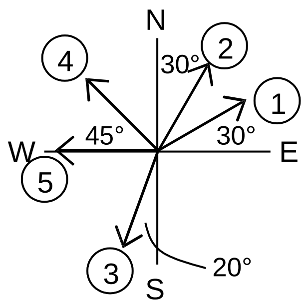
- The north-south line of a compass is considered the reference line and **all directions** are given in terms of an angle **measured clockwise from north**.
- North is considered to be 0° or 360°, with east at 90°, south at 180° and west at 270°.
- When **bearing** is used, it is always expressed as **three numbers**.
- In the following example, the directions of the vectors are 045° (not 45°), 135°, 225° and 315° respectively.



Interesting! In aviation, pilots apparently prefer to use 360° for north and not 0° (000°). Is this true, and if, why would that be?

2 Quick facts: Compass directions as a way to describe the direction of a vector

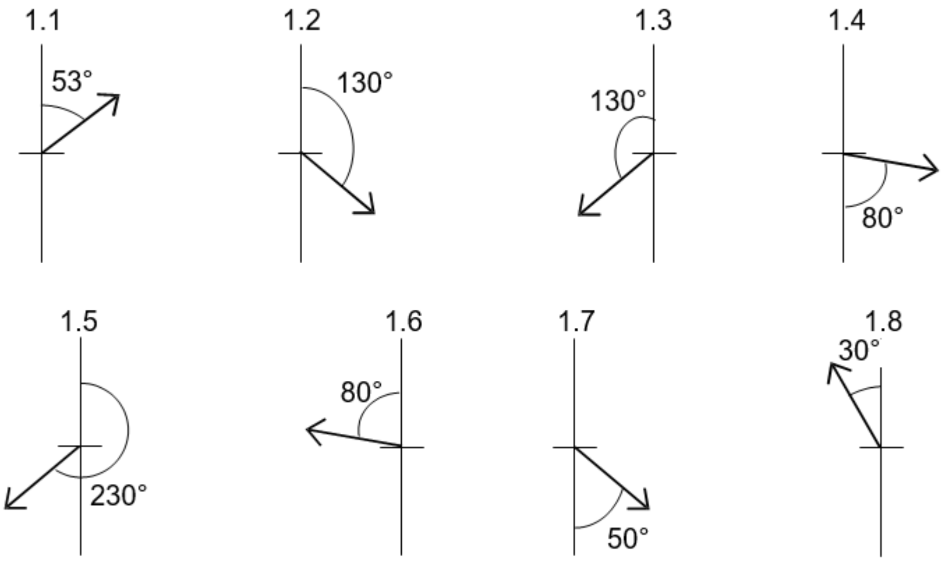
- A vector's direction can be described as an angle of rotation about its tail from the main compass directions (or points) of north, east, south and west as in the example.
  - Vector 1 30° north of east, abbreviated as E30°N; this means you face east and then rotate towards the north through an angle of 30°.
  - Vector 2 30° east of north, abbreviated as N30°E; face north, then rotate towards the east through an angle of 30°.
  - Vector 3 20° west of south or S20°W
  - Vector 4 45° north of west or W45°N.
  - Vector 5 This vector's direction is referred to as only "west" or "due west".



Below is an Activity for you to test you understanding of vectors, bearing and compass direction. Some of the answers are provided so you can test if you have mastered this section. Redo those which you got wrong and make sure that you revisit the examples above.

Activity 1.1

1. Describe the direction of each of the following vectors in terms of bearing and in terms of compass directions. The vertical line represents the north-south axis.

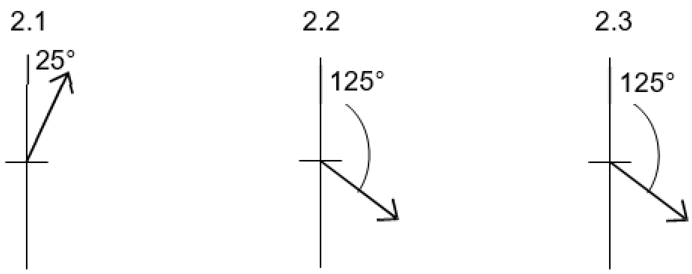


You should get the following answers. Refer to the Quick Facts if you are still uncertain.

1.1	053°	N53°E (53° east of north)	E37°N (37° north of east)
1.2	130°	E40°S (40° south of east)	S50°E (50° east of south)
1.3	230°	W40°S (40° south of west)	S50°W (50° west of south)
1.4	100°	S80°E (80° east of south)	E10°S (10° south of east)
1.5	230°	S50°W (50° west of south)	W40°S (40° south of west)
1.6	280°	N80°W (80° west of north)	W10°N (10° north of west)
1.7	130°	S50°E (50° east of south)	E40°S (40° south of east)
1.8	330°	N30°W (30° west of north)	W60°N (60° north of west

2. Draw diagrams such as in the previous question, with correctly measured angles, to represent the following vectors
   - 2.1 025°
   - 2.2 S55°E
   - 2.3 125°

The following diagrams are not necessarily accurate, but they give an idea of the correct answer.



B. Graphs

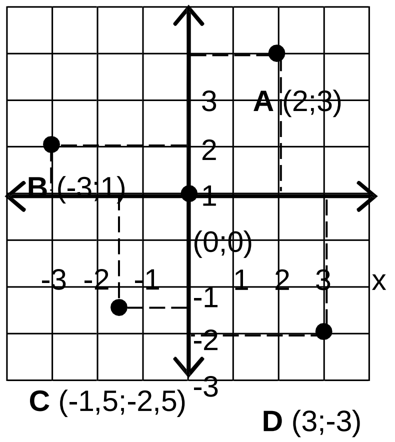
- Demonstrate direct proportion graphs in the context of technology.
  - Recall that straight line graphs are represented by $y = mx + c$.
  - Recall that hyperbolic graphs are represented by $xy = k$.
- Demonstrate inverse proportion graphs in the context of technology.

In science we are interested in the relationship between numbers, e.g. when we do experiments and investigations. During this course in grade 11 we are interested in direct proportionalities and inverse proportionalities (Note: Do not refer to "indirect" proportionality for the latter.)

1. Quick facts on Cartesian Plane

- The Cartesian plane is a plane surface with an x-axis (horizontal) and a y-axis (vertical) intersecting at right angles
- Horizontally to the right of the origin are the positive x-values and horizontally to the left the negative x-values. Positive y-values are above the origin on the vertical axis and negative y-values below the origin.
- The two axes divide the Cartesian plane in four quadrants.

  Points **A**, **B**, **C** and **D** in the diagram are in quadrant 1, 2, 3 and 4 respectively.



Any point in the Cartesian plane can be uniquely described using one pair of coordinates.

- **Coordinates have x-values and y-values, in this order, separated by a semicolon (;)**.
- In the diagram, the origin is represented by (0;0). Points **A**, **B, C** and **D** are represented by (2;3), (-3;1), (-1,5;-2,5) and (3;-3) respectively.

Direct Proportional

Inverse Proportional

2 Quick facts: Direct proportionality

- You want to buy a jigsaw for your workshop. If the price of one jigsaw is R500, then it is easy to determine that two, three and four jigsaws cost R1 000, R1 500 and R2 000 respectively, if no discount is involved.

The data can be recorded as follows:

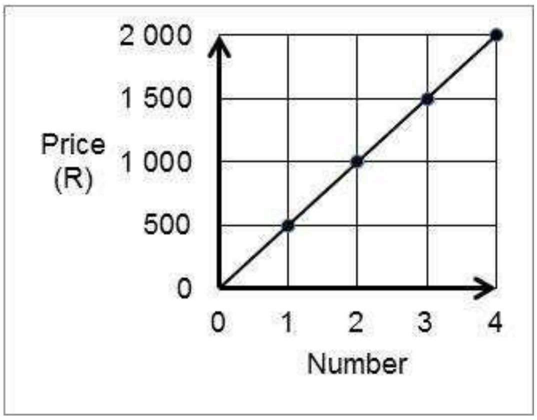
Number of jigsaws	Price (R)
1	500
2	1 000
3	1 500
4	2 000

A C D B

- **A direct proportion implies that two sets of numbers increase or decrease by the same factor**.

A From 1 to 4 it is an increase of four times.
B The respective numbers of 500 to 2 000 also increase four times.
C 2 is one half of 4
D 1 000 is also one half of 2 000
.
Hence, in any of these cases, if you divide the price by the number of jigsaws, you get the same answer.

A graph for this data is as follows:



- The price depends on the number of jigsaws; hence price is the **dependent variable**; number is the **independent variable**. They are plotted on the y- and x-axes respectively.
- The gradient of any graph is the quotient of the "change in y-coordinates" divided by "change in x-coordinates". In symbols:

  gradient = $\frac{\Delta y}{\Delta x}$
- Any two sets of coordinates can be used to calculate the gradient.

- For this graph: gradient = $\frac{\Delta \text{price}}{\Delta \text{number}} = \frac{1500 - 1000}{3 - 2} - 500 \text{ rand per jigsaw}$
- Any straight-line graph can be described with the following equation:

$$\boxed{y = \overbrace{m}^{\text{gradient}}x + \overbrace{c}^{\text{y-intercept}}}$$

The y-intercept is where it intersects the y-axis.

- For the graph above, the equation is: y = 500x + 0 or y = 500x.

- From this, if x = 4, y = 500x = (500)(4) = 2 000. Any other value of x can be used to calculate a value of y, even if x and y are not on the graph. The equation describes the relationship between x and y for all values.

- This is possible because the graph **intersects the origin**.

- If the y-intercept is **not in the origin**, it is **NOT** a direct proportionality.

Time to practise !

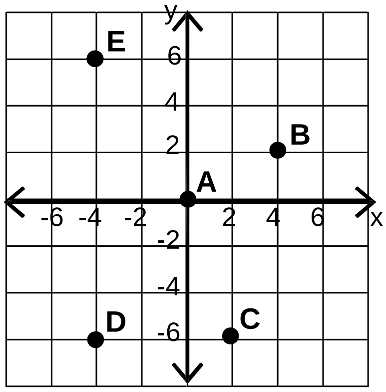
Activity 1.2 below is an opportunity for you to test your understanding.

You will notice a progression in the type of questions. Be patient and start at the top as a guarantee that you have mastered the basics and that you will be able to answer the more advanced questions on a particular graph as you will then need to interpret the information in order to determine whether you are presented with a direct or inverse proportionality.

You should also be able to construct a graph from data presented in a table.
Take special notice of the Cartesian plane depicted to assist you with the concept scale.

Acitivity 1.2

1. Consider the following graph paper with points plotted on it.



1.1 Which one of points **A** to **E** is plotted at the origin?
1.2 Give the coordinates of point **B**.
1.3 What are the x-coordinates of points **C** and **D**?
1.4 What are the y-coordinates of points **D** and **E**?

1.5 Consider a line that goes through points **D** and **B**.
1.5.1 Calculate the gradient of the line.
1.5.2 Calculate the y-intercept of the line.
1.5.3 Determine the equation of the line.

1.6. Repeat question 1.5, but for a line that goes through points **B** and **E**.
1.7 Repeat question 1.5, but for a line that goes through points **E** and **D**.
1.8 Repeat question 1.5, but for a line that goes through points **D** and **C**.
1.9 Can you see a pattern between the orientation of each line and its gradient in questions 1.5 to 1.8?
1.10 Calculate the y-coordinate of the line that goes through points **E** and **B** for x = 9.
1.11 Calculate the y-coordinate of the line that goes through points **D** and B for x = 9.

2. Consider the following table with x and y values for three points **P**, **Q** and **R**.

	x	y
P	1	1
Q	2	2
R	3	3

2.1 Plot points **P**, **Q** and **R** on graph paper.
2.2 Draw the best-fit line through these three points.

2.3 Use your graph to determine the:
2.3.1 y-value for x = 1,5
2.3.2 x-value for y = 3,5

2.4 What is the relationship between the x and y values? Write down your answer in words as well as in symbols.
2.5 Predict the y-value for x = 150.

2.6 The formula that represents a straight line is y = mx + c.
2.6.1 What is represented by the "m"?
2.6.2 Use an appropriate formula and determine the value of m for your graph.
2.6.3 What is represented by the "c"?
2.6.4 Determine the value of c.
2.6.5 Write down the equation for your graph in the form of y = mx + c.

Some of the solutions to Activity 1.2 are given below. Always refer back to the notes when you are in doubt. DO NOT FORGET YOUR SIGN FOR NEGATIVE VALUES.

You need to make sure that you can determine the gradient of a straight line graph as this can be tested in most experiments or whenever you deal with relationships between variables.

Activity 1.2

1.1 A
1.2 (4; 2)
1.3 2 & -4
1.4 -6 & 6

1.5
1.5.1 For coordinates D (-4;-6) and B (4; 2):

$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{2-(\ \ 6)}{4-(-4)} -1$$

1.5.2 For D (-4;-6):
y = mx + c
-6 = 1(-4) + c
∴ c = -2

1.5.3 y = x - 2

1.6
1.6.1 For coordinates B(4;2) and E(-4;6):

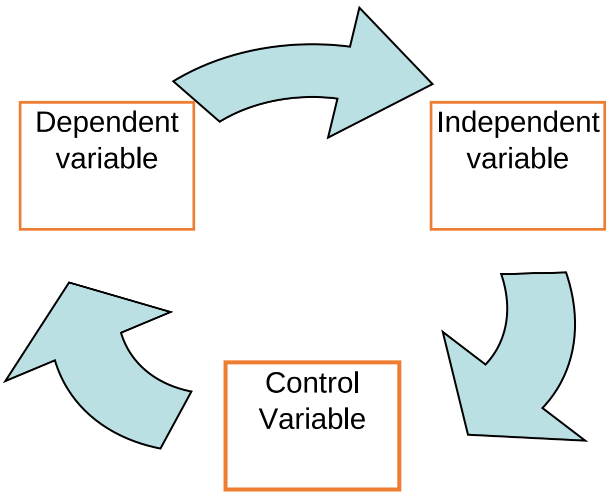
$$\text{Gradient} = \frac{\Delta y}{\Delta x} = \frac{2-6}{4-(-4)} = -0,5$$

1.6.2 For E (-4;6):
y = mx + c
6 = (-0,5)(-4) + c
∴ c = 4

1.6.3 y = -0,5x + 4

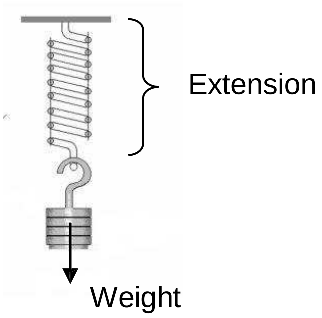
Activity 1.3 test the same content, but with application in Technology.

Revisit the terms Dependent-, Independent- and Control variables.



Activity 1.3

1. An experiment is done to investigate the behaviour of a spring. The extension of the spring is measured for an increasing number of mass pieces that are attached to it. All the mass pieces have identical weights and one mass piece has a weight of 5 N. The following set of data is obtained.



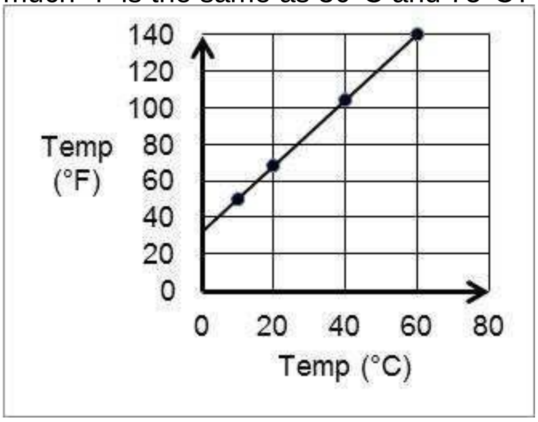
Weight (N)	Extension (mm)
10	5
20	10
30	15
40	20

1.1 Which one of weight or extension is the dependent variable?
1.2 Draw a graph of these results by plotting weight on the x-axis and extension on the y-axis.
1.3 What is the relationship between extension and weight for this spring? Give a reason for your answer.
1.4 Determine the gradient of the graph.
1.5 Determine the equation for this graph.
1.6 How many mass pieces cause an extension of 15 mm?

2. You want to "design" a formula to convert temperatures measured in °C to °F. You use two thermometers, one calibrated in °C and the other in °F, to take the temperature of water in a beaker. After that you heat the water slightly and take the temperature again. You repeat this process for a further two sets of readings and record your results as follows.

Temp (°C)	Temp (°F)
10	50
20	68
40	104
60	140

2.1 By inspecting the numbers, would you say this is a direct proportionality? Give a reason for the answer.
2.2 Based on the pattern in the table, can you tell how much °F is the same as 30°C and 75°C?
2.3 As a good scientist, you decide to draw a graph for these results and obtain the following.



2.3.1 By inspecting the graph, would you say this is a direct proportionality? Give a reason for the answer.
2.3.2 Determine the gradient of the graph.
2.3.3 Determine the y-intercept of the graph.
2.3.4 Determine a formula that describes this graph.
2.3.5 Use your formula to determine how much °F is the same as 30°C and 75°C.
2.3.6 What is the boiling point of water in °F at sea level if the boiling point at sea level is 100°C.

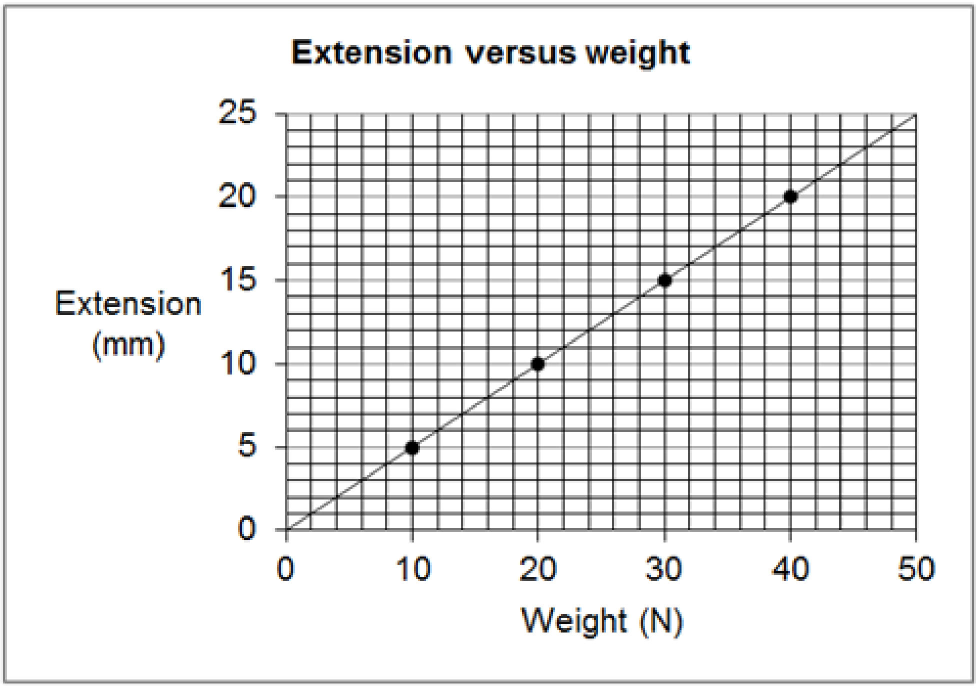
Skill required : Constructing a straight line graph 10

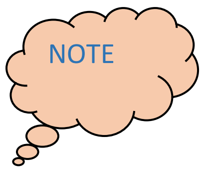
The answer to 1.2 is given below.

Constructing a straight line graph is a very important skill that you need to master. Practise, practise, practice.

Step 1: Identify the variables.
Step 2: Determine the variable range.
Step 3: Determine the scale of the graph.
Step 4: Number and label each axis and title the graph.
Step 5: Determine the data points and plot on the graph.
Step 6: Draw the graph.

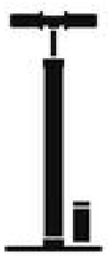
Remember to use a sharp pencil.





The title of the graph must include the variables. You need to refer to a relationship between the variables.

1.2.3 Quick facts: inverse proportionality



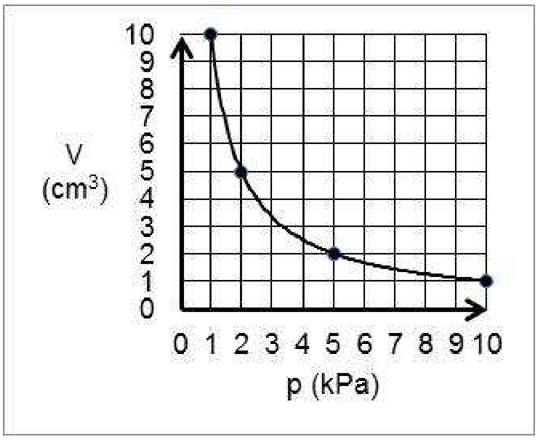
- You know that when you use a pump to inflate your bicycle-tires, the volume of air inside the cylinder of the pump decreases as you press down on the plunger, i.e. when you increase the pressure. Typical results for this are displayed in the table.

Pressure (kPa)	Volume (cm^3)
1	10
2	5
5	2
10	1

A B

- **An inverse proportion implies that one set of numbers increase with the same factor as the decrease of another set of numbers**.
  A From 2 to 10 it is an increase of a factor five.
  B When 5 decrease with a factor 5, it becomes 1

- A graphs for this type of data is as follows:



- Volume depends on pressure. Volume is the **dependent** variable (on y-axis) and pressure is the **independent** variable (on x-axis).
- A graph like this can be described with the following equation:

$$\boxed{xy = k}$$ constant

- The graph never intersects the axes; it strives towards the axes, but does not intersect them.
- This type of graph is called a hyperbola.

The activity below tests your understanding of Inverse proportionality.
The answers to 1.1 to 1.5 are given.
You attempt 2.1 to 2.6 on your own.
Always refer back to the Quick facts when in doubt.

You have covered speed and velocity in Grade 10. The formula for speed is given in the activity. You should be familiar with it.

Activity 1.4

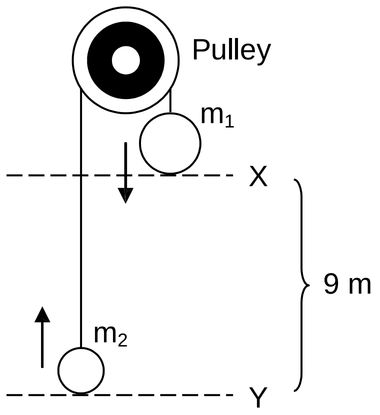
1. You plan a trip to Cape Town and want to get an idea of the relationship between your average speed and travelling time. The distance of your journey will be 1 200 km.

Use $\boxed{\text{average speed} = \frac{\text{distance}}{\text{time}}}$ to calculate the missing information in the following table.

Average speed ($km \cdot h^{-1}$)	Time (h)
75	
100	
150	
160	

1.1 Complete the table and draw a graph of your results (if you do not have graph paper, just design simple graph paper yourself). Put average speed on the x-axis.
1.2 Identify the dependent and independent variables.
1.3 What type of relationship is this? Give a reason for your answer.
1.4 Use your graph to read off your average speed if you travel for 576 minutes.
1.5 Use a calculation to test your answer to the previous question.

2. The diagram represents an Atwood machine, which is a simple device used to study motion. In this case, it consists of a frictionless pulley and two mass pieces, m_1 and m_2, which are connected by a light, inelastic rope. The mass pieces are held stationary at positions **X** and **Y**; m_1 is **heavier** than m_2. When the mass pieces are released, you measure the distance moved by each mass piece **from X**. Your values **for m_1** (called distance 1) are displayed in the following table:



Distance 1 of m_1 from X (m)	Distance 2 of m_2 from X (m)
1	
4	
9	

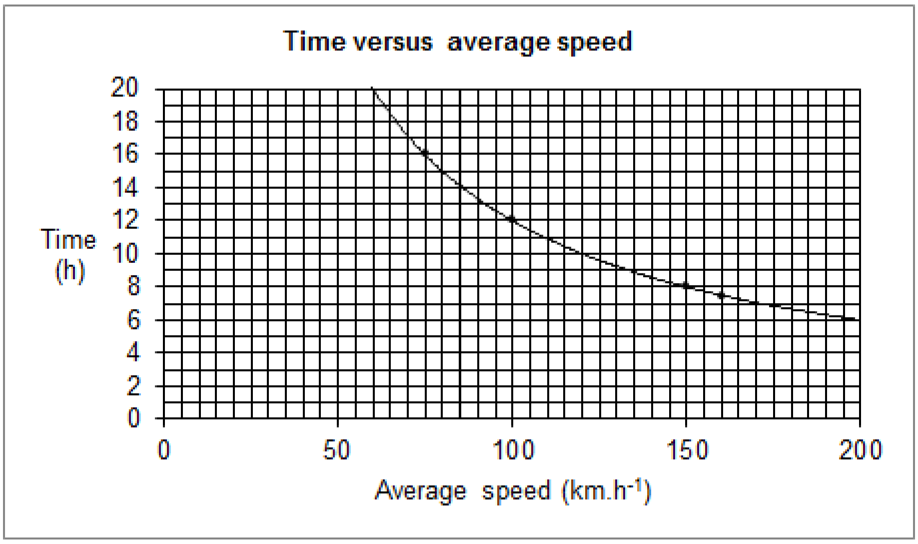
2.1 By analysing the diagram, complete the corresponding values for distance 2 **from X**.
2.2 Which one of distance 1 or distance 2 is the dependent variable?
2.3 Draw a piece of simple graph paper and plot this data.
2.4 Is this an example of an inverse proportionality? Give a reason for your answer.
2.5 Use your graph to read off the value for distance 2 if distance 1 is equal to 5 m.
2.6 Determine the equation of your graph and test your answer to the previous question.

Activity 1.4

Solutions for Questions 1.1 to 1.5

1.1

Average speed	Time (h)
75	16
100	12
150	8
160	7,5



1.2 Dependent variable : time

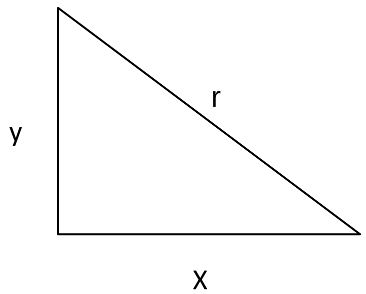
Independent variable: average speed

1.3 Inverse proportionality.
One set of numbers increase with the same factor as the decrease of the other set of numbers OR the product of each set of numbers gives a constant value.

1.4 125 km·h^{-1}

1.5 $\text{speed} = \dfrac{\text{distance}}{\text{time}} = \dfrac{1\,200}{9{,}6} = 125\ \text{km}\cdot\text{h}^{-1}$

C Quick facts: theorem of Pythagoras



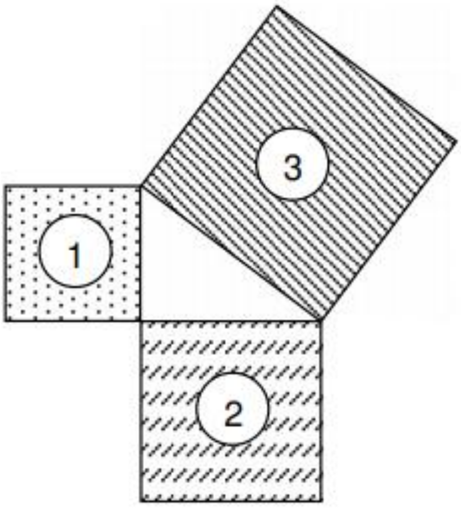
- Pythagoras was born in 570 BC.
- He became one of the most famous Greek mathematicians and philosophers and is inter alia known for his theorem of Pythagoras.

The Theorem of Pythagoras:
For any right- angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

- In symbols: $r^2 = x^2 + y^2$
- **Example**: If x and y are equal to 4 and 3 respectively, the length of r van be calculated as follows:

  $r^2 = x^2 + y^2 = 4^2 + 3^2 = 25. \quad \therefore r = 5.$

- You can visualise this by using the following diagram, where the area on each side of the triangle is related in the following way: Area of 3 is equal to the sum of the areas of 1 and 2.



TOPIC 2: VECTORS

The content below indicates the content that you should master in order to be successful in control tests and examinations.

Collinear and coplanar vectors

- Define collinear vectors as vectors that have the same line of action.
- Define coplanar vectors as vectors that are in the same plane.
- Draw the resultant of two collinear vectors.

Resultant of forces in two dimensions

- Use scale drawings to determine the resultant of vectors as indicated below.
- Use the tail-to-head method (also called head-to-tail) to determine the resultant of two vectors at right angles to each other. *This will be expanded to include two vectors which are acting at other angles than 90° to each other.*
- Use the theorem of Pythagoras to determine the resultant of two forces acting at right angles to each other. *This will include simple trigonometry.*
- State and understand the parallelogram law of forces.
- Use the parallelogram of forces to determine the resultant of two forces at any angle to each other (no calculations when the angle is not a right angle).

Experiment 1: Parallelogram of forces

- Use the parallelogram of forces to determine the:
  - Resultant of two forces acting on a point
  - Weight of a given body
- *This will be expanded to include the:*
  - *Equilibrant and its relationship with the resultant*
  - *Relationship between the vector diagram obtained through the tail-to-head method and three forces in equilibrium*

Resolution of a force into its components

- Use scale drawings and calculations to resolve a force F, which is acting at an angle to the horizontal, into its rectangular components (components parallel and perpendicular to the horizontal axis).

Important terms/definitions

Collinear vectors	Vectors that have the same line of action.
Components of a force.	Forces, when acting together, have the same effect on a body as the original force. (This is true in general for all vectors.)
Coplanar vectors	Vectors that are in the same plane.
Equilibrant	A single force that keeps other forces in equilibrium. It has the same magnitude as the resultant force, but acts in the opposite direction.
Equilibrium	The resultant of the forces acting on an object is equal to zero.
Parallelogram law for forces	The parallelogram law for forces states that if two forces that are acting at the same point is represented by the adjacent sides of a parallelogram both in magnitude and direction, the diagonal from the point gives the resultant of the two forces.
Resultant	The vector sum of two or more vectors, i.e. a single vector having the same effect as two or more vectors together.
Vector addition	Add vectors by taking into account magnitude and direction.

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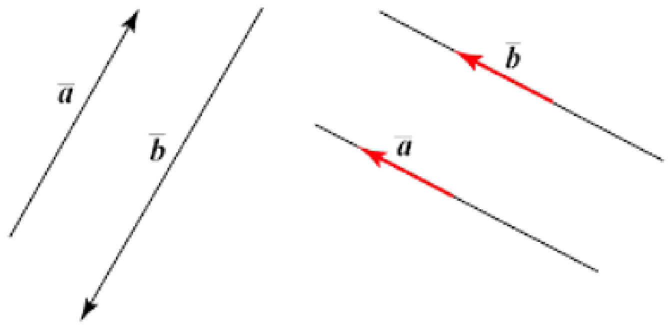
2.1 Quick facts: collinear vectors

- **Collinear vectors** are defined as **vectors that have the same line of action**.



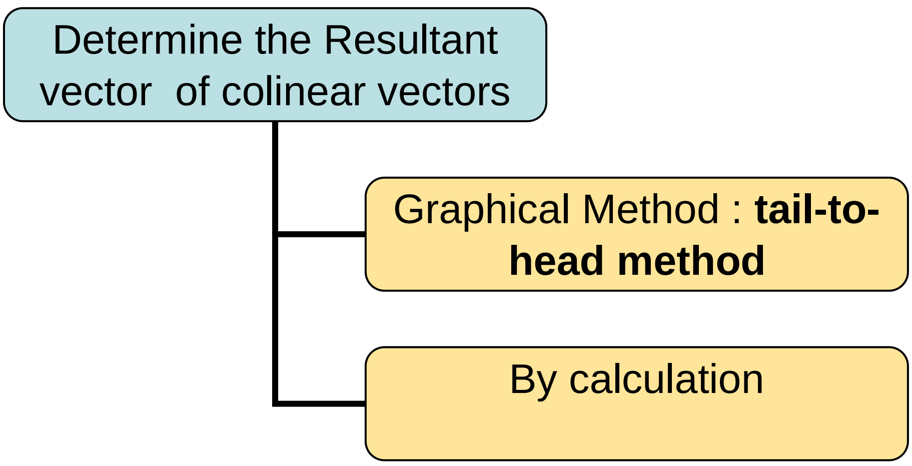
From this, the following:

- The vectors are parallel to one another.
- They either have the same direction or opposite directions.
- The angle between them is either 0° or 180°.
- We can say they are in a one-dimensional (1D) configuration.

Note

- When one direction is chosen as positive, the other direction is obviously negative.

- The **resultant**
A single vector having the same effect as two or more vectors together. Another term for "resultant" is "net".
The vector sum of two or more vectors



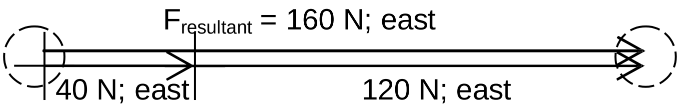
- Summary of the graphical method: Tail-to-head method (or head-to-tail method
  - To get the resultant, draw an arrow from the tail of the first vector to the head of the last vector. The tail of the arrow representing the resultant must touch the tail of the first vector and its head must touch the head of the last vector.
  - Measure the direction/angle of the resultant.
  - Measure the length of the resultant and use the scale to convert it back to the required magnitude.
  - Ensure that you label all the vectors with their magnitudes and directions.

Example:

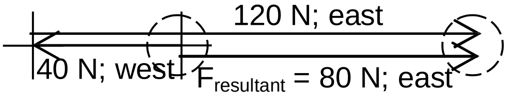
If you exert a force of 120 N on an object in an easterly direction, and your friend exerts a force of 40 N on the same object in the same direction, the construction to determine the resultant force (or net force) is as follows [in the constructions below, the vectors are slightly displaced so that you can clearly see how the construction is done]:

Note which tails and which heads are together to get the resultant. Never draw the dotted line circles; here it is just for understanding.

Scale: 10 mm: 20 N



If the 40 N force is due west, the construction, using the same scale, is as follows:



- Steps for the **calculation method:**
  - Draw a rough vector diagram to help you analyse the situation.
  - Choose a positive direction. Then the opposite direction is obviously negative.
  - Determine the resultant by adding the vectors in a way that we call **vector addition**. This implies that you take into account not only magnitude, but also direction.
  - Once you have the answer, express it in terms of magnitude and direction.

  - **Example**: The first example above is solved like this:

Choose east as positive.

$F_{resultant} = F_1 + F_2$
$= (+120) + (+40)$
$= +160$ N
$\therefore F_{resultant}$ = 160 N; east

or

Choose west as positive.

$F_{resultant} = F_1 + F_2$
$= (-120) + (-40)$
$= -160$ N
$\therefore F_{resultant}$ = 160 N; east

Note that in both cases the final interpretation of the answer is the same.

Notice sign convention

  - **Example**: The second example is solved like

Choose east as positive.

$F_{resultant} = F_1 + F_2$
$= (+120) + (-40)$
$= +80$ N
$\therefore F_{resultant}$ = 80 N; east

or

Choose west as positive.

$F_{resultant} = F_1 + F_2$
$= (-120) + (+40)$
$= -80$ N
$\therefore F_{resultant}$ = 80 N; east

Note that in both cases the final interpretation of the answer is the same.

Activity 2.1

1. Determine the resultant vector in each of the following cases by means of a scale drawing. Test each of your answers by means of a calculation.

1.1 A displacement of 25 m, bearing of 010° and another displacement of 25 m; bearing 190°.
1.2 15 N north and 19 N south.
1.3 12,8 km in the direction E10°S and 14,2 km in the same direction.
1.4 20 km in a direction of W20°N and 30 km in the opposite direction.
1.5 10 N to the left, 2 N to the right, 6 N to the right and 5 N to the right.

2. Draw the forces in the previous sub-question in the opposite order; i.e. start with the 5 N force and end with the 10 N force.

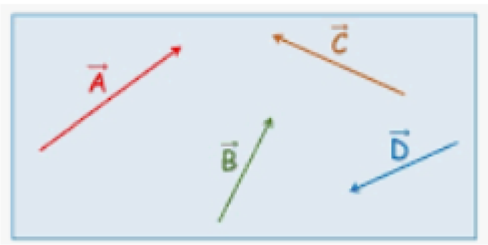
2.1 Is the answer the same as before?
2.2 What can you conclude from that?

3. Consider two forces of 20 N and 15 N.

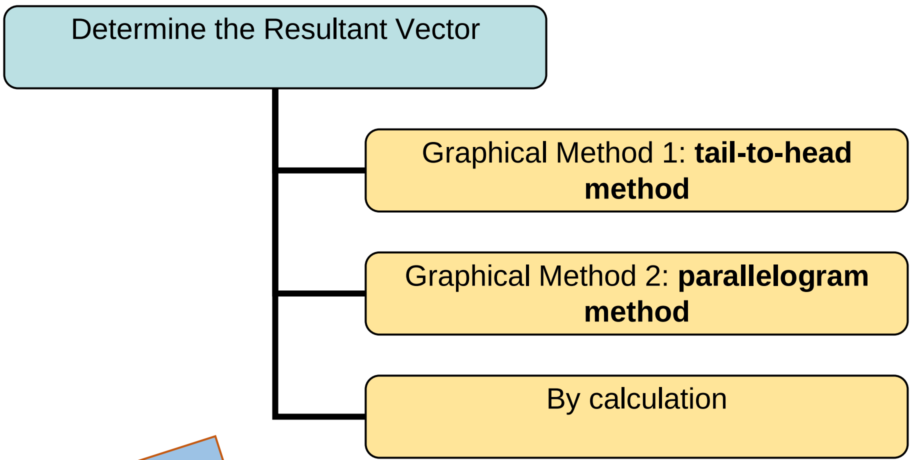
3.1 What is magnitude of the maximum resultant?
3.2 What is the angle between the forces to give a maximum resultant?
3.3 What is magnitude of the minimum resultant?
3.4 What is the angle between the forces to give a minimum resultant?

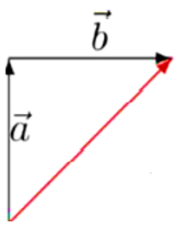
2.2 Quick facts: Coplanar vectors

- **Coplanar vectors** are defined as **vectors that are in the same plane**.



  - The angle between the vectors is anything other than 0° or 180°.
  - We can say they are in a two-dimensional (2D) configuration.





Note

.
The **tail -to-head method** is used in the same way as for collinear vectors. The difference is that the vectors are not acting in the same line anymore; hence you will get a <u>triangle</u>, which is formed by the two vectors and the resultant vector.

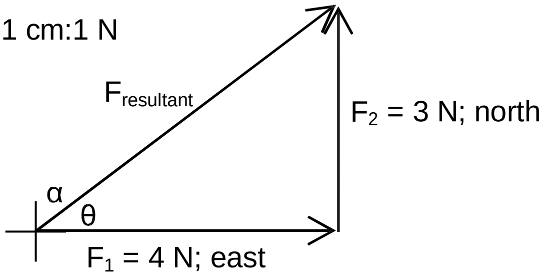
The parallelogram law for forces

If two forces that are acting at the same point are represented in magnitude and direction by the adjacent sides of a parallelogram, the diagonal from the point gives the resultant of the two forces.

- **Example**: Two horizontal forces, F_1 = 4 N east and F_2 = 3 N north, are acting on a box. Determine the resultant of these two forces on the box.

- The **tail-to-head method**:

Scale: 1 cm:1 N

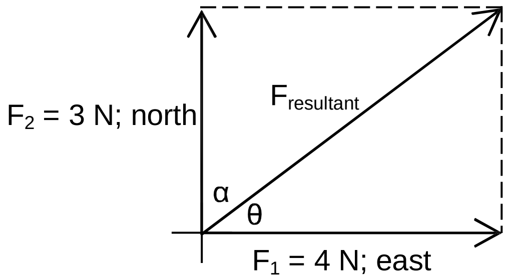


If α is measured, the direction can be expressed in terms of bearing, i.e. 053°. Direction can also be given by measuring θ and express it as E37°N.

$F_{resultant}$ = 5 N; see block for explanation of direction.

- The **parallelogram method** (which gives a rectangle in this case, because the angle between F_1 and F_2 is 90°):

Scale: 1 cm:1 N



$F_{resultant}$ = 5 N; directions are used as for the tail-to-head method.

- The two methods demonstrated above can also be used to determine the resultant of two forces with any angle between them.

- The **calculation method** (only required for two vectors with a 90° angle between them):
  - Draw a rough vector diagram to help you analyse the situation.
  - Use the theorem of Pythagoras and simple trigonometry for right-angled triangles to determine the resultant force.
  - The trigonometric formulae, referring to the sides of a right-angled triangle, are:

$\text{Sin } \Theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	$\cos \Theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	$\text{Tan } \Theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$$\begin{aligned} F^2_{resultant} &= F_1^2 + F_2^2 \\ &= 4^2 + 3^2 \\ &= 25 \end{aligned}$$

$$\tan\theta = \frac{3}{4}$$

$$\theta = 36{,}87^\circ$$

$$F_{resultant} = \sqrt{25} = 5\,\text{N}$$

$\therefore F_{resultant}$ = 5 N; 053,13° (for example)

Activity 2.2

1. A car travels a distance of 10 km due north and then it travels a distance of 8 km at a bearing of 090°. Determine the resultant displacement of the car with respect to its point of departure by using:

1.1 The tail-to-head method
1.2 The parallelogram method
1.3 A calculation

2. A river flows from east to west. A man in a rowing boat steers the boat perpendicular to the opposite bank of the river, which is 45 m wide. By the time he reaches the opposite bank, the water has moved him and his boat 90 m downstream. Determine his resultant displacement by using:

2.1 The tail-to-head method
2.2 The parallelogram method
2.3 A calculation

3. Two tractors are pulling a crate, each one exerting a horizontal force of 700 N on the crate. The angle between the forces is 30°.

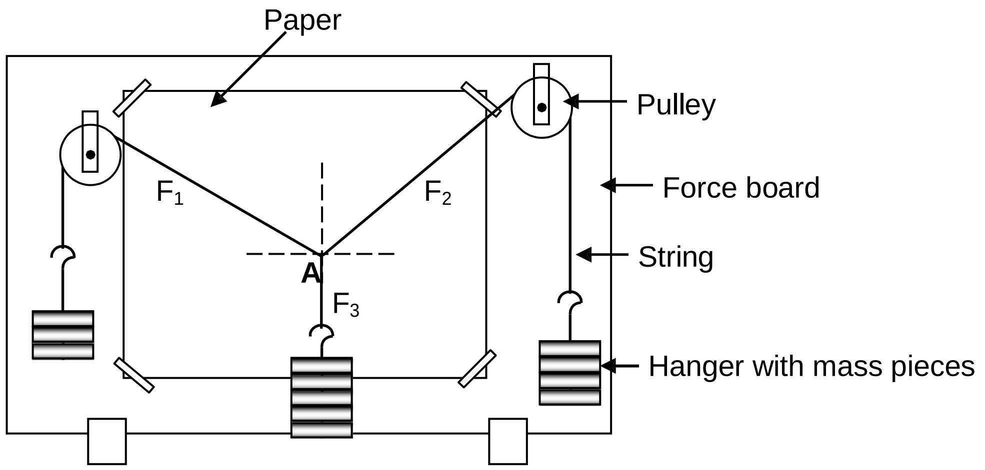
3.1 Determine the resultant force on the crate by using the tail-to-head method and the parallelogram method.
3.2 What happens to the magnitude of the resultant force if the angle is:
3.2.1 Less than 30°
3.2.2 More than 30°

4. The two tractors are pulling another crate by simultaneously exerting horizontal forces on the crate. The one tractor exerts a force of 550 N in the direction 090° while the other tractor exerts a force of 550 N in the direction W45°N. Determine the resultant force by using the tail-to-head method and the parallelogram method.

Experiment 1: Three forces in equilibrium

Aim
To investigate the effect of three forces acting at a point that is **in equilibrium**.

Apparatus
Force board apparatus (pulleys, hangers, mass pieces, string, etc.)

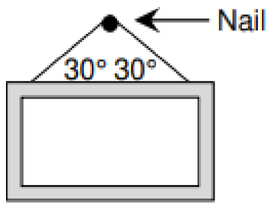


You have performed this experiment and you should have gained a lot of insight and strengthening in your understanding of using the parallelogram law of forces to determine the resultant force.

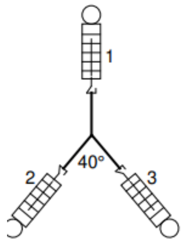
Apply this strengthening to work through the questions in Activity 2.3.

Activity 2.3

1. A painting is supported by a string of which the ends make angles of 30° with the top of the painting as indicated. Use a construction to determine the magnitude of the upward force by the nail on the string if the force exerted by each string is 10 N.



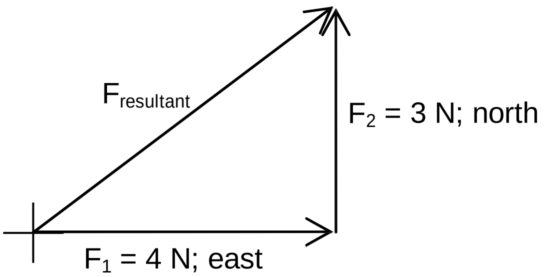
2. Three spring-balances are connected by strings as shown. The angle between the strings of spring-balances 2 and 3 is 40°. Spring-balance 1 keeps the other two in equilibrium. If the reading on each of 2 and 3 is equal to 16 N, determine by means of construction and measurement the reading on spring-balance 1.

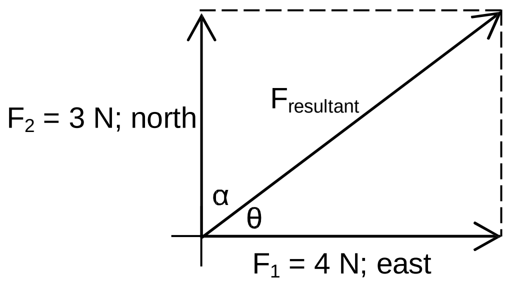


Lesson 7 discusses the Parallelogram Law of forces in great detail. Do revisit that particular lesson if you still struggle. Step by step directions are provided.

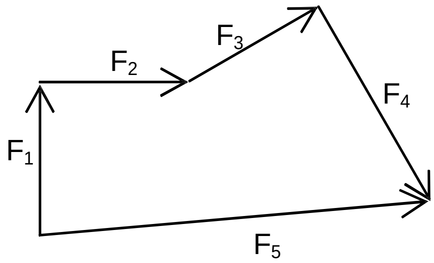
2.3 Quick facts: resolution of forces in their components

- In previous sections you have learned how to determine the resultant vector of two or more other vectors.
- Refer to the following constructions you have used to determine the resultant force of forces F_1 and F_2. In vector terminology, **F_1 and F_2** are called the **components of $F_{resultant}$**.





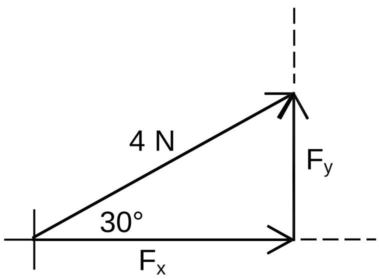
- Any vector can have an infinite number of components, as you can see from the following vector diagram, which is drawn using the tail-to-head method. F_1 to F_4 are drawn tail to head, but the tail of F_5 touches the tail of F_1 (first vector); the head of F_5 touches the head of F_4 (last vector). Therefore, F_1 to F_4 **are the components** of F_5, and F_5 is **the resultant** of F_1, F_2, F_3 and F_4.



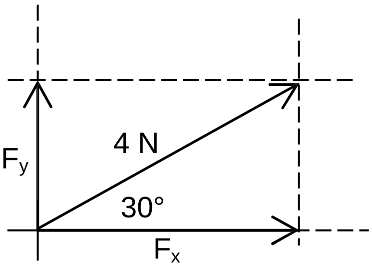
- We are not interested in so many components of a vector. We want to restrict components to those that are perpendicular to each other, and furthermore we want to restrict components to horizontal and vertical ones only. This means we want to have x- and y-components of a vector.
- You can determine components **graphically** and by means of **calculations**.
- When you use the graphical method, the techniques you have learned about in sections 2.1 and 2.2 are still valid, although determining components of a vector is doing the opposite than determining the resultant vector of the components.

- **Example**: A force of 4 N is applied to an object at an angle of 30° to the positive x-axis. Resolve this force into its horizontal and vertical components.

  - For the **tail-to-head method**:
    - Choose a scale.
    - Draw the cross-hairs.
    - Construct the 4 N force; in magnitude and direction.
    - Draw a horizontal line from the tail of the 4 N force. This will become F_x; the horizontal component.
    - Drop a perpendicular from the head of the 4 N force. This will become F_y; the vertical component.
    - Put in the arrow heads. Tail of F_x touches tail of 4 N; head of F_x touches tail of F_y; head of F_y touches head of 4 N.
    - Measure the lengths of F_x and F_y. Use the scale to convert them to forces. In this case F_x = 3,5 N and F_y = 2 N



- For the **parallelogram method**:



  - Choose a scale.
  - Draw the cross-hairs.
  - Construct the 4 N force; in magnitude and direction.
  - Draw a horizontal line from the tail of the 4 N force. This will become F_x; the horizontal component.
  - Draw a vertical line from the tail of the 4 N force. This will become F_y; the vertical component.
  - Draw a horizontal line from the head of the 4 N force, parallel to F_x until it intersects the vertical line you have drawn from the cross-hairs.
  - Drop a vertical from the head of the 4 N force, parallel to F_y until it intersects the horizontal line you have drawn from the cross-hairs.
  - Put in the arrow heads as shown in the diagram. All the tails are together.
  - Measure the lengths of F_x and F_y. Use the scale to convert them to forces. In this case F_x = 3,5 N and F_y = 2 N.

- The **calculation method**:

  - Draw a rough vector diagram to help you analyse the situation.
  - Like before, use the theorem of Pythagoras and simple trigonometry for right-angled triangles to determine the components. In this example:

$$\begin{aligned} \sin 30^\circ &= \frac{F_y}{4} \\ F_y &= (4)\sin 30^\circ \\ &= 2\,\text{N} \end{aligned} \qquad\qquad \begin{aligned} \cos 30^\circ &= \frac{F_x}{4} \\ F_x &= (4)\cos 30^\circ \\ &= 3{,}46\,\text{N} \end{aligned}$$

AKNOWLEDGEMENT:

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